Oversampling Architecture for Analog Harmonic Modulation

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Abstract—We propose a technique for sinusoidal modulation of analog signals, which requires neither multiplication nor analog sine wave generation. The modulating signal is an oversampled representation of a sine in the form of a periodic binary sequence, optimized to minimize intermodulation distortion. A 2-to-1 multiplexer performs the modulation multiplication. With third-order lowpass filtering of the input signal, the technique achieves 60 dB of linear dynamic range for a sequence of length 256, conveniently generated with an 11-address decoder and an 8-bit counter.

1 Introduction

Besides the widespread use of modulation and demodulation in communication systems, harmonic modulators and demodulators are used frequently in signal processing systems, such as subband coders and lock-in amplifiers. Sinusoidal modulation and demodulation of an input time-varying signal $x(t)$ consist of multiplying the input by a sinusoidal reference signal $s(t) = \sin(\omega_0 t + \phi_0)$ and filtering the product to remove the sum components (demodulation) or difference components (modulation). The general system is shown in Figure 1(a). The output is described by

$$y(t) = (x(t) \cdot s(t)) * h(t), \tag{1}$$

where $*$ denotes the convolution operator, and $h(t)$ is the impulse response of the lowpass filter.

The difficulty of implementing such a system in analog circuitry is the need for a highly linear multiplier and a distortion-free sinusoid of precise frequency and amplitude. Often a bank of identical demodulators is required, as for subband coding [1]. Correspondingly, the design must be compact, with uniform characteristics across different instances.

We propose a system implementation derived from mixing digital and analog techniques. We show that this system is equivalent to the system described above except for a small, controllable degree of distortion. The proposed system, however, requires neither a high-precision multiplier nor an analog sine wave, and can be built in compact form with conventional technologies.

2 Multiplexing vs. Multiplying

The modulation system which we propose is shown in Figure 1(b). The multiplier is replaced by a lowpass prefilter, a unity-gain inverter, and a multiplexer [2]. The modulating sinusoid $s(t)$ is replaced by a binary

(± 1) sequence $s'(t)$ which determines whether the multiplexer passes the prefiltered input or its complement to its output.

The result is effectively a multiplication of the prefiltered input with the sequence $s'(t)$. The key is to choose the sequence such that the result closely approximates modulation with the sinusoid $s(t)$.

We note that this architecture is analogous to FIR filters with oversampled binary encoding of the coefficients and/or the input signal [3, 4]. Both systems rely on the filtering function inherent in the implemented system to serve the additional purpose of filtering out the noise produced by the process of oversampling.

The new system is described by

$$y(t) = \left[(x(t) * g(t)) \cdot s'(t) \right] * h(t), \tag{2}$$

with $g(t)$ the impulse response of the lowpass prefilter. The binary sequence is assumed periodic with quarter-wave symmetry, and therefore can be described in the frequency domain by a Fourier series containing only harmonics of odd orders in ω_0:

$$S'(\omega) = \sum_{k=0}^{\infty} S'_k(\omega)\,, \tag{3}$$

$$S'_k(\omega) = jc_k \left[\delta\left(\omega - (2k+1)\omega_0\right) - \delta\left(\omega + (2k+1)\omega_0\right) \right]. \tag{4}$$

The fundamental component $S'_0(\omega)$ corresponds to the sinusoidal signal $s(t)$.

Figures 2(a) and (b) show how the two systems operate under the assumption that the lowpass filter functions $H(\omega)$ and $G(\omega)$ are ideal, *i.e.*, flat in the passband and with infinite rolloff at the cutoff frequency. Under this assumption, it can be seen from Figure 2 that an arbitrary input spectrum $X(\omega)$ corresponding to the time-domain input $x(t)$ produces the same output $y(t)$ for both systems if and only if the prefilter bandwidth BW($G(\omega)$) is constrained by

$$\omega_0 + \mathrm{BW}(H(\omega)) < \mathrm{BW}(G(\omega)) < 3\omega_0 - \mathrm{BW}(H(\omega))\,. \tag{5}$$

If the last inequality is not satisfied, convolution products of $X(\omega) * S'_k(\omega)$ for $k > 0$ will be aliased into the output.

In reality, the filters $H(\omega)$ and $G(\omega)$ have finite rolloffs, and the equivalence between the systems in Figure 1 is only approximate. The quality of the approximation depends on the harmonic coefficients c_k corresponding to the binary sequence, which can be optimized for minimum distortion. There is a trade-off between the complexity of the sequence and that of the filters G and H, as illustrated in Section 4.

3 Sequence generation

The output spectrum generated by the modulation scheme contains intermodulation products between the prefiltered input $G(\omega) \cdot X(\omega)$ and the harmonics of $S'_k(\omega)$, with terms of the form

$$jc_k\, G\left(\omega \pm (2k+1)\omega_0\right) \cdot X\left(\omega \pm (2k+1)\omega_0\right) \cdot H(\omega)\,. \tag{6}$$

Only the fundamental term $k = 0$ is desired, and distortion arises from the higher-order intermodulation products, $k > 0$. To reduce distortion, the coefficients c_k need to be small for $k > 0$, except for large values of k for which the terms of (6) are reasonably small due to the attenuation by the prefilter G. In other words, the low-frequency components of the binary sequence $s'(t)$ need to approximate the sine wave $s(t)$ as closely as possible. Qualitatively, this corresponds to an oversampled noise-shaped sine wave, as produced by a delta-sigma modulator [5].

Techniques for deriving periodic sequences with several zero or small harmonic components of c_k are presented in [6]. We formulate the problem of finding an optimal binary sequence directly from a minimum distortion criterion on the intermodulation components (6).

In general, the amount of distortion is input dependent, and assumptions need to be made on $X(\omega)$ to formulate an optimization criterion. Our criterion is to maximize the ratio of energy in the fundamental harmonic modulation component ($k = 0$) to the combined energy of the distortion components ($k > 0$). Assuming a narrow bandwidth of $H(\omega)$ and an input spectrum $X(\omega)$ which in the worst case is flat in amplitude, the criterion becomes:

$$\text{Maximize :} \quad \frac{c_0^2 |G(\omega_0)|^2}{\sum_{k=1}^{\infty} c_k^2 |G((2k+1)\omega_0)|^2} \ . \tag{7}$$

which is equivalent to minimizing the harmonic distortion of the sequence $s'(t)$, filtered with the same prefilter $G(\omega)$. The criterion can be applied, in principle, to select the optimal bit sequence $s'(nT)$, for $n = 1 \ldots N$. With quarter-wave symmetry, only $N/4$ bits (one quadrant) need to be determined. Still, this problem has a combinatorial complexity, and becomes intractable for large N. We obtain approximate solutions using a technique of iterative block optimization, where a full search is repeatedly conducted over randomly selected blocks of consecutive bits in the sequence.

4 Results and implementation

We demonstrate the principle with the following example, which offers a simple and elegant implementation. The filter $G(\omega)$ is third order, implemented as a cascade of three single-pole filter stages, each pole located at $z = 15/16$. The sequence length is $N = 256$, and the iterative block optimization method (with block size 8) has been performed to obtain the 64 bits in the first quadrant of the sequence, shown in Figure 3(a). The FFTs of the filtered and unfiltered binary sequences are shown in Figure 3(b). All harmonics of the filtered sequence are more than 60 dB below the fundamental. The magnitude of the prime harmonic of the sequence is approximately 1.02, or within 2% of unity.

The advantages of using an oversampled modulation sequence rather than a simpler binary sequence can be appreciated by following comparison. To obtain the same 60 dB linearity performance with, say, a square wave modulator, a premultiplication filter G with much sharper rolloff such as a fourth-order Chebyshev or a 6th-order Butterworth would be required to compensate for the sizable harmonics in the square wave. Such a filter would be more complicated to build than the simple cascade of single-pole filters in the above example, and would be more sensitive to mismatches in the implementation. On the other hand, the oversampled sequence is fairly easy to generate, as outlined below.

The first-quadrant sequence of Figure 3(a) consists of 11 zeros and 53 ones. This asymmetry allows a simple implementation of an address decoder locating the eleven "zero" bits. The process of reversing and inverting the sequence, needed to obtain the other three quadrants of the sequence, is simplified by using a gray code counter to produce the addresses. In an eight-bit gray code, the lower six bits are a reversing sequence, and the seventh bit controls the inversion of the sequence through an exclusive-or operation. It is also straightforward to generate the addresses of a sequence 90° out of phase with the original, for complex modulation with both sine and cosine components.

5 Conclusions

We have designed an architecture which efficiently and accurately implements complex harmonic modulation/demodulation of an analog signal. The system function approximates multiplication by a sinusoid, with a high degree of linearity and low distortion, without the need of precise analog components. In place of an analog multiplier, the implementation uses a lowpass prefilter, unity-gain inverter, and a 2-to-1 analog multiplexer controlled by a periodic bit sequence. The sequence is efficiently generated by a gray-code counter and a sparse address decoder.

References

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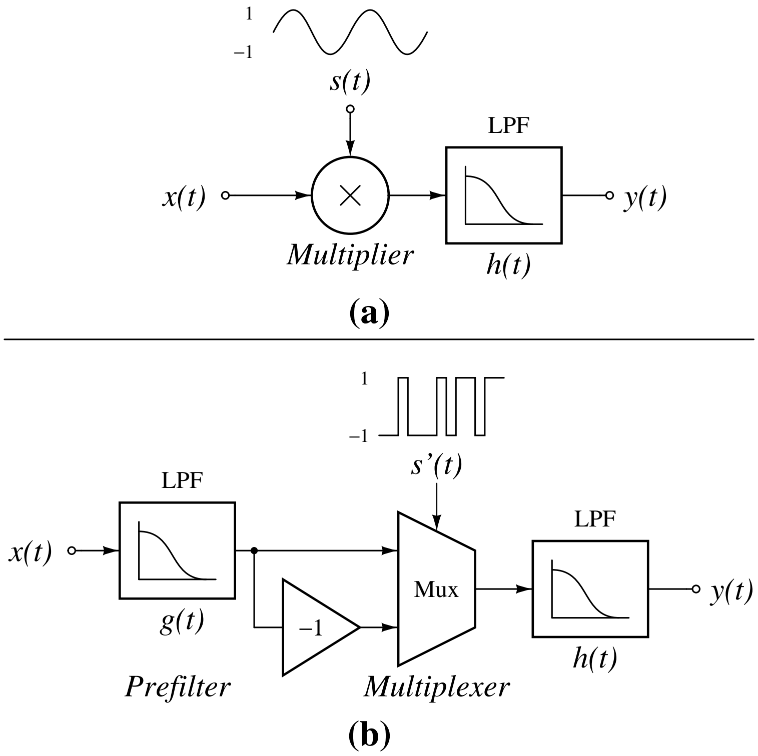

Figure 1: *Demodulation system, using multiplication or multiplexing.*

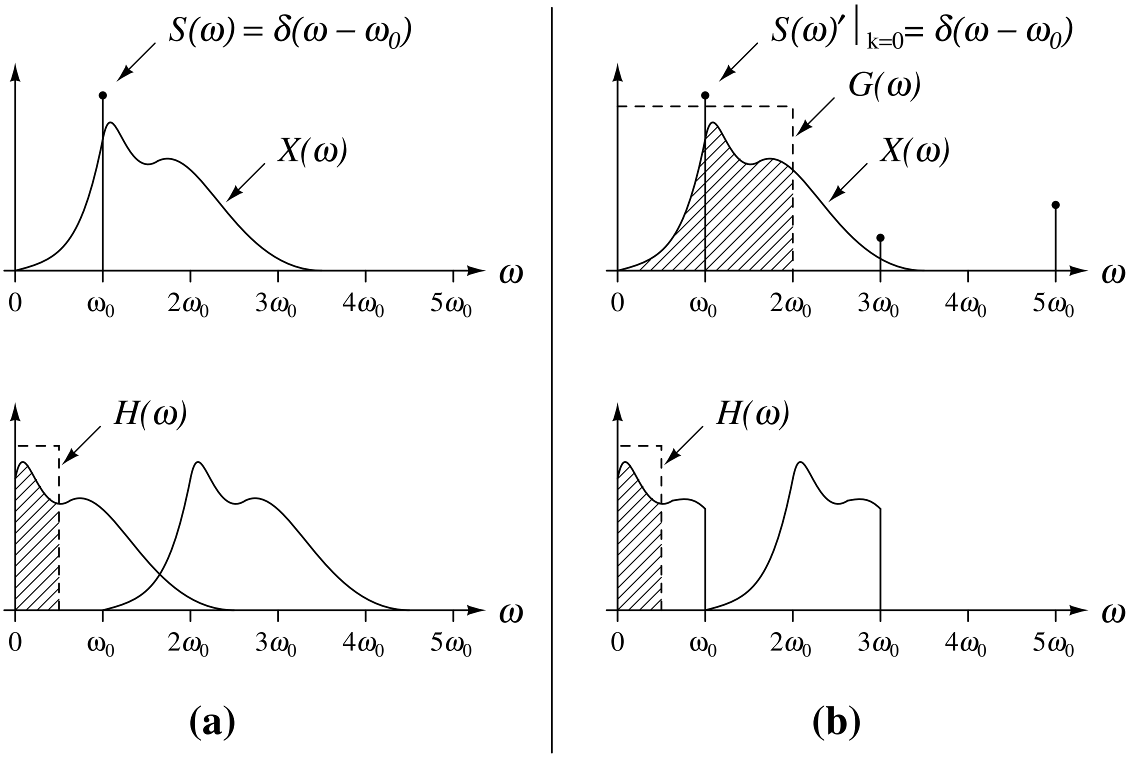

Figure 2: *Demodulation process with systems of Figure 1.*

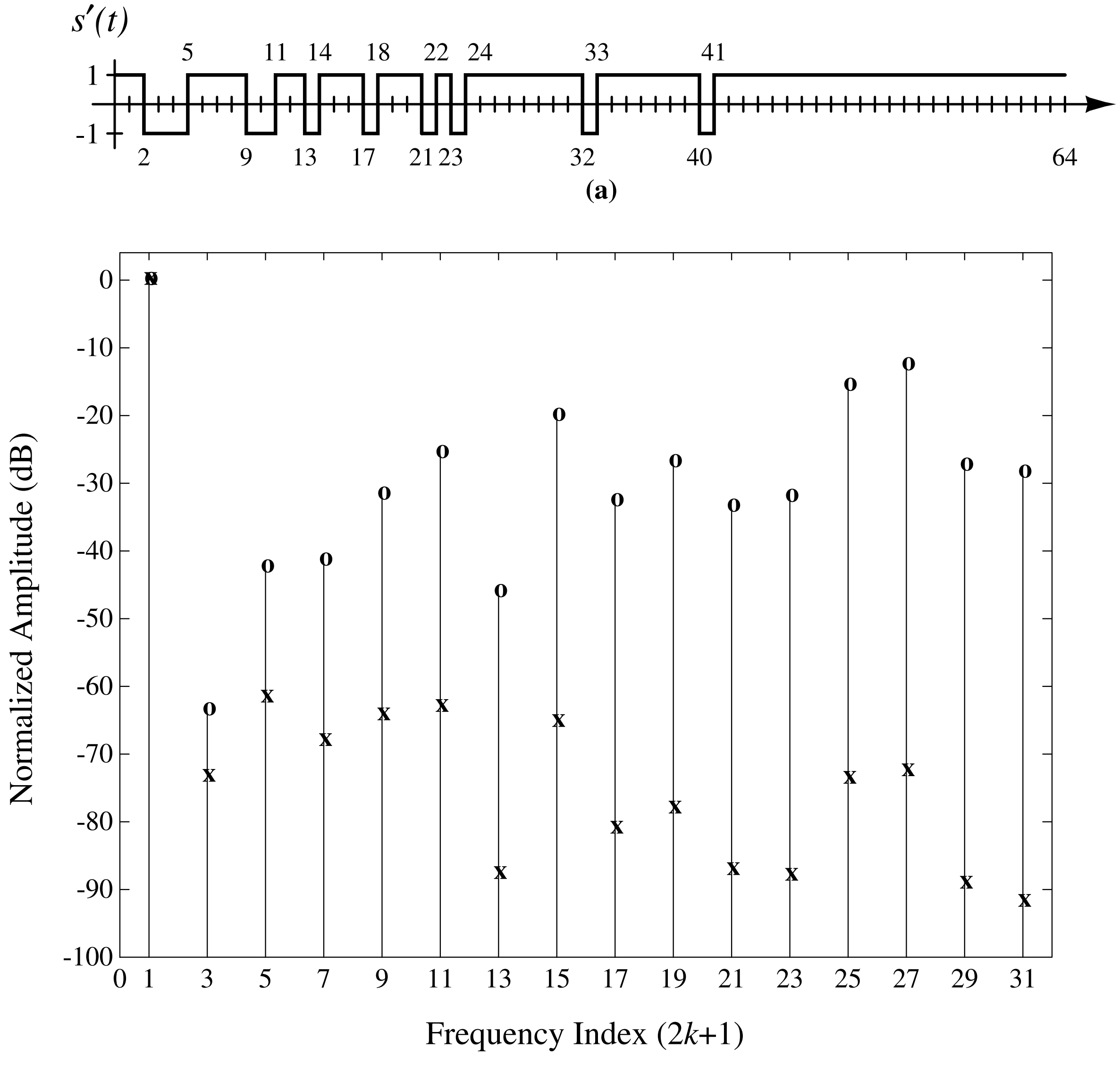

Figure 3: **(a)** *Optimized 64-bit oversampled sine sequence, first quadrant;* **(b)** *Frequency domain properties of the raw (***o***) and filtered (***x***) bit sequences.*